CSE 150A-250A AI: Probabilistic Models

Lecture 10

Fall 2025

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Slides adapted from previous versions of the course (Prof. Lawrence, Prof. Alvarado, Prof Berg-Kirkpatrick)

Review

EM Application

Hidden Markov Models

Test 1

A. Excellent 5%

B. Pretty good. 12%

C. So-so ... 50%

D. Blah

E. :c 20%

Review

ML estimation for complete data

- Notation

  Nodes $X_1, X_2, \ldots, X_n$
  Examples $t = 1, 2, \ldots, T$
  Complete data $\{(x_{1t}, x_{2t}, \ldots, x_{nt})\}_{t=1}^{T}$

- ML estimates for CPTs

root nodes

$$P_{\mathrm{ML}}(X_i = x) = \frac{\mathrm{count}(X_i = x)}{T} = \frac{1}{T}\sum_t I(x_{it}, x)$$

nodes with parents

$$P_{\mathrm{ML}}(X_i = x | \mathrm{pa}_i = \pi) = \frac{\mathrm{count}(X_i = x, \mathrm{pa}_i = \pi)}{\mathrm{count}(\mathrm{pa}_i = \pi)} = \frac{\sum_t I(x_{it}, x)\, I(\mathrm{pa}_{it}, \pi)}{\sum_t I(\mathrm{pa}_{it}, \pi)}$$

ML estimation for incomplete data

- **Notation**

  Nodes $X_1, X_2, \ldots, X_n$
  Examples $t = 1, 2, \ldots, T$
  Visible nodes $V_t = v_t$ for t^{th} example

- **EM algorithm**

  Initialize CPTs to nonzero values.
  Repeat until convergence:

  E-step — compute posterior probabilities.
  M-step — update CPTs:

root nodes	$$P(X_i = x) \longleftarrow \frac{1}{T} \sum_t P(X_i = x \mid V_t = v_t)$$
nodes with parents	$$P(X_i = x \mid \text{pa}_i = \pi) \longleftarrow \frac{\sum_t P(X_i = x, \text{pa}_i = \pi \mid V_t = v_t)}{\sum_t P(\text{pa}_i = \pi \mid V_t = v_t)}$$

Complete versus incomplete data

- Complete data

root nodes	$P_{\mathrm{ML}}(X_i = x) \quad = \quad \frac{1}{T} \sum_t I(x_{it}, x)$
nodes with parents	$P_{\mathrm{ML}}(X_i = x \vert \mathrm{pa}_i = \pi) \quad = \quad \frac{\sum_t I(x_{it}, x)\, I(\mathrm{pa}_{it}, \pi)}{\sum_t I(\mathrm{pa}_{it}, \pi)}$

- Incomplete data

root nodes	$P(X_i = x) \quad \longleftarrow \quad \frac{1}{T} \sum_t P(X_i = x \vert V_t = v_t)$
nodes with parents	$P(X_i = x \vert \mathrm{pa}_i = \pi) \quad \longleftarrow \quad \frac{\sum_{t=1} P(X_i = x, \mathrm{pa}_i = \pi \vert V_t = v_t)}{\sum_{t=1}^{T} P(\mathrm{pa}_i = \pi \vert V_t = v_t)}$

Key properties of EM

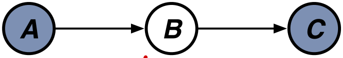
- No learning rate

  The updates do not require the tuning of a learning rate ($\eta > 0$), as in purely gradient-based methods.

- Monotonic convergence

  Changes to CPTs from the EM updates always increase the incomplete-data log-likelihood $\mathcal{L} = \sum_t \log P(V_t = v_t)$.

EM Example



Incomplete data $\{(a_t, c_t)\}_{t=1}^{T}$
A and C are observed.
B is hidden.

- E-step (Inference)

$$P(b|a_t, c_t) = \frac{P(c_t|b)\, P(b|a_t)}{\sum_{b'} P(c_t|b')\, P(b'|a_t)}$$

- M-step (Learning)

$$P(a) = \frac{1}{T}\,\mathrm{count}(A = a)$$

$$P(b|a) \longleftarrow \frac{\sum_t I(a, a_t)\, P(b|a_t, c_t)}{\sum_t I(a, a_t)}$$

$$P(c|b) \longleftarrow \frac{\sum_t I(c, c_t)\, P(b|a_t, c_t)}{\sum_t P(b|a_t, c_t)}$$

EM Application

Application

- Statistical language modeling

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  Let w_ℓ denote the ℓ^{th} word in a corpus of text.
  How to model $P(w_1, w_2, \ldots, w_L)$?

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model	$P(w_1, w_2, \ldots, w_L)$	ML estimate	DAG

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model	$P(w_1, w_2, \ldots, w_L)$	ML estimate	DAG
unigram	$\prod_\ell P_1(w_\ell)$	$P_1(w) = \frac{\text{count}(w)}{L}$	$\boxed{w_1}$ $\boxed{w_2}$ $\cdots$ $\boxed{w_L}$

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bigram	$\prod_\ell P_2(w_\ell \vert w_{\ell-1})$	$P_2(w' \vert w) = \frac{\text{count}(w \to w')}{\text{count}(w)}$	$\boxed{w_1} \to \boxed{w_2} \to \cdots \to \boxed{w_L}$

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- Evaluating n-gram models

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- Evaluating n-gram models

  Train on corpus $\mathcal{A}$ $\quad \Longrightarrow \quad$ $P_1(\mathcal{A}) \le P_2(\mathcal{A}) \le P_3(\mathcal{A}) \ldots$

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- Evaluating n-gram models

$$\text{Train on corpus } \mathcal{A} \quad \Longrightarrow \quad P_1(\mathcal{A}) \leq P_2(\mathcal{A}) \leq P_3(\mathcal{A}) \ldots$$
$$\text{Test on corpus } \mathcal{B} \quad \Longrightarrow \quad P_2(\mathcal{B}) = 0 \text{ if } \mathcal{B} \text{ has unseen bigrams.}$$

Word clustering

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- Alternative to bigram model

Word clustering

- **Alternative to bigram model**

  Insert a hidden node $Z \in \{1, 2 \ldots, C\}$ between the previous and next words $W, W' \in \{1, 2, \ldots, V\}$.

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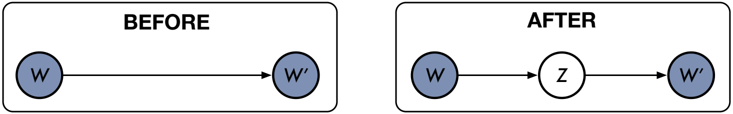

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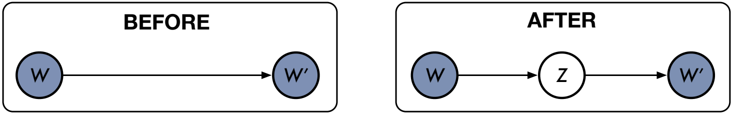

  Words W and W' are observed (as before).
  The node Z is a latent variable to detect word clusters.

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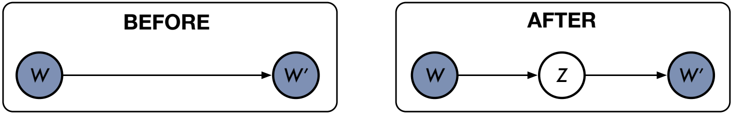

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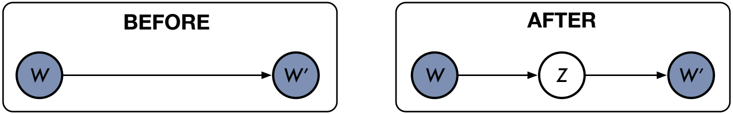

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  $P(z|w)$ is the probability that word w is mapped into cluster z.

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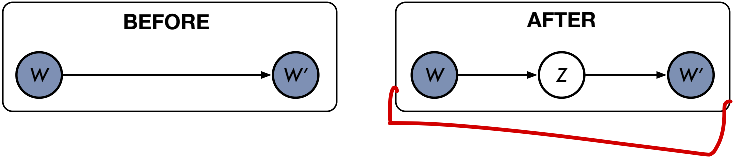

  Words W and W' are observed (as before).
  The node Z is a latent variable to detect word clusters.

- **Conditional probability tables**

  $P(z|w)$ is the probability that word w is mapped into cluster z.
  $P(w'|z)$ is the probability that word w' follows any word in cluster z.

Computing $P(w'|w)$

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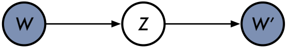
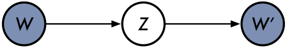
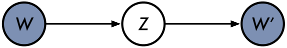


Computing $P(w'|w)$



- Inference

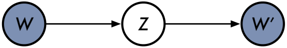
Computing $P(w'|w)$



- Inference

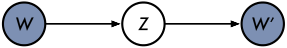
$$P(w'|w)$$

Computing $P(w'|w)$



- Inference

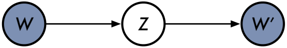
$$P(w'|w) = \sum_z P(w', z|w)$$



- Inference

$$P(w'|w) = \sum_z P(w', z|w) \qquad \boxed{\text{marginalization}}$$

Computing $P(w'|w)$

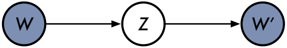


- Inference

$$\begin{aligned} P(w'|w) &= \sum_z P(w', z|w) \\ &= \sum_z P(w'|z, w)\, P(z|w) \end{aligned}$$

marginalization

Computing $P(w'|w)$

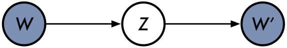


- Inference

$$P(w'|w) = \sum_z P(w', z|w) \quad \text{marginalization}$$

$$= \sum_z P(w'|z, w)\, P(z|w) \quad \text{product rule}$$

Computing $P(w'|w)$



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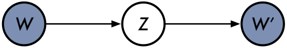
Computing $P(w'|w)$



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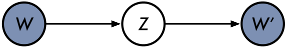
Computing $P(w'|w)$



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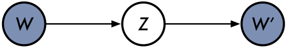
- Matrix factorization

Computing $P(w'|w)$



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- Matrix factorization

The above expresses the matrix $\overbrace{P(w'|w)}^{V\times V}$ as the product of the two smaller matrices $\underbrace{P(w'|z)}_{V\times C}$ and $\underbrace{P(z|w)}_{C\times V}$.

Model complexity

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- Parameter count

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size of vocabulary	V

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size of vocabulary	V
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Model complexity

- **Parameter count**

size of vocabulary	V	
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parameters in cluster model	$2CV$	$P(w'\|z), P(z\|w)$

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parameters in bigram model	V^2	$P(w'\|w)$

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parameters in unigram model	V	$P(w)$

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- Compact representations of complex worlds

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- Compact representations of complex worlds

  Setting $C=1$, we recover the unigram model.

Model complexity

- **Parameter count**

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number of clusters	C	
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- **Compact representations of complex worlds**

Setting $C = 1$, we recover the unigram model.
Setting $C = V$, we recover the bigram model.

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- Compact representations of complex worlds

Setting $C = 1$, we recover the unigram model.
Setting $C = V$, we recover the bigram model.
In between, we are exploring a range of different models.

EM algorithm

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The model is the same as our previous example.

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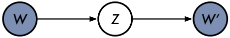
Only the variable names have changed!



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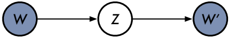

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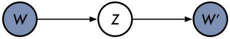

- **E-step – Inference**

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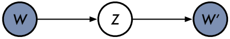

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$$P(z|w_\ell, w_{\ell+1}) \; = \; \frac{P(w_{\ell+1}|z)\, P(z|w_\ell)}{\sum_{z'} P(w_{\ell+1}|z')\, P(z'|w_\ell)}$$

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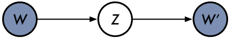

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- M-step – Learning

EM algorithm

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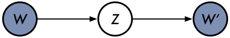

- E-step – Inference

$$P(z|w_\ell, w_{\ell+1}) = \frac{P(w_{\ell+1}|z)\, P(z|w_\ell)}{\sum_{z'} P(w_{\ell+1}|z')\, P(z'|w_\ell)}$$

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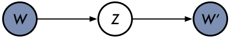

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Post

EM algorithm

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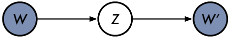

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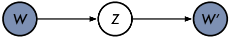

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Experimental results

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- Data set

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  60K-word vocabulary

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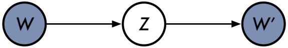
- Model 

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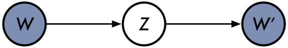

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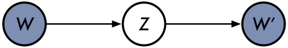
- **Model** 

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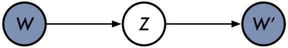

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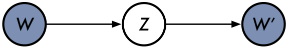
- **Model** 

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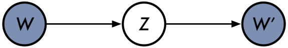
- **Model** 

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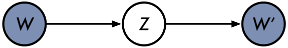
- **Model** 

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- **Results**

  The model has no prior knowledge of word meanings.
  Which words does it cluster? Look at $\mathrm{argmax}_z\, P(z|w)$.

Word clusters

Word clusters

1	as cents made make take
2	ago day earlier Friday Monday month quarter reported said Thursday trading Tuesday Wednesday ⟨...⟩
3	even get to
4	based days down home months up work years ⟨%⟩
5	those ⟨,⟩ ⟨—⟩
6	⟨.⟩ ⟨?⟩
7	eighty fifty forty ninety seventy sixty thirty twenty ⟨(⟩ ⟨·⟩
8	can could may should to will would
9	about at just only or than ⟨&⟩ ⟨;⟩
10	economic high interest much no such tax united well
11	president
12	because do how if most say so then think very what when where
13	according back expected going him plan used way
15	don't I people they we you
16	Bush company court department more officials police retort spokesman
17	former the
18	American big city federal general house military national party political state union York
19	billion hundred million nineteen
20	did ⟨"⟩ ⟨'⟩
21	but called San ⟨:⟩ ⟨start-of-sentence⟩
22	bank board chairman end group members number office out part percent price prices rate sales shares use
23	a an another any dollar each first good her his its my old our their this
24	long Mr. year
25	business California case companies corporation dollars incorporated industry law money thousand time today war week ⟨)⟩ ⟨unknown⟩
26	also government he it market she that there which who
27	A. B. C. D. E. F. G. I. L. M. N. P. R. S. T. U.
28	both foreign international major many new oil other some Soviet stock these west world
29	after all among and before between by during for from in including into like of off on over since through told under until while with
30	eight fifteen five four half last next nine oh one second seven several six ten third three twelve two zero ⟨-⟩
31	are be been being had has have is it's not still was were
32	chief exchange news public service trade

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8	can could may should to will would
9	about at just only or than ⟨&⟩ ⟨;⟩
10	economic high interest much no such tax united well
11	president
12	because do how if most say so then think very what when where
13	according back expected going him plan used way
15	don't I people they we you
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The table shows the most likely cluster assignments $\text{argmax}_z P(z|w)$ for the 300 most common tokens in the corpus.

Example : Noisy-OR

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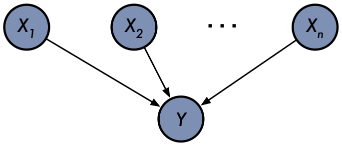

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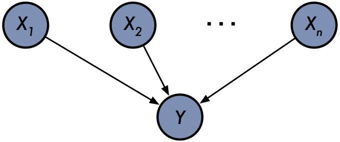

$x_i \in \{0, 1\}$

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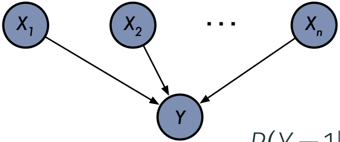

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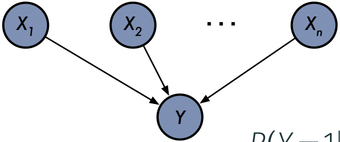

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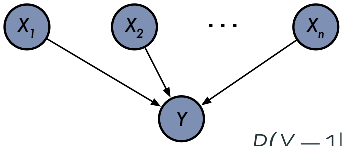

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How to estimate parameters $p_i \in [0, 1]$ that maximize this?

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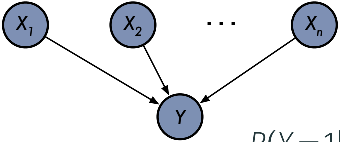

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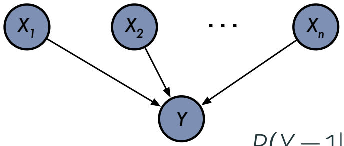

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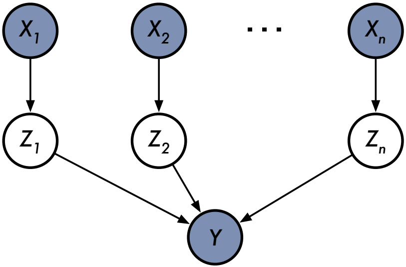
EM — but how? Isn't the data complete?

EM for noisy-OR



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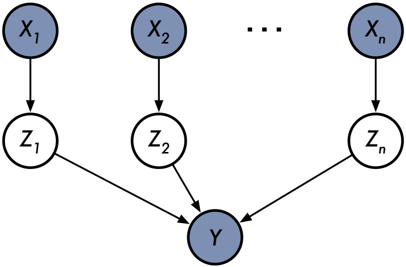

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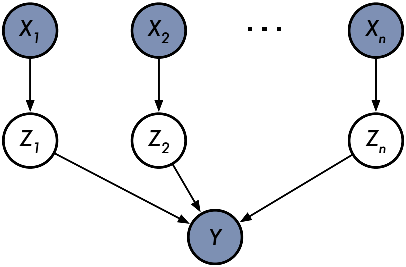

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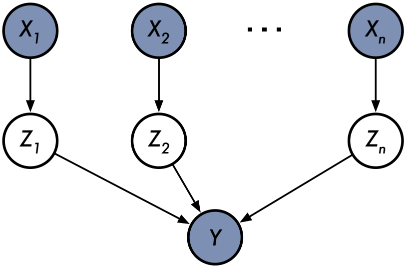

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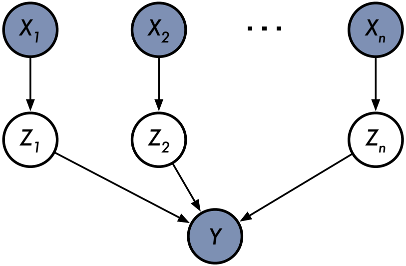

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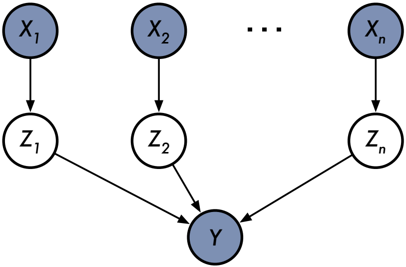

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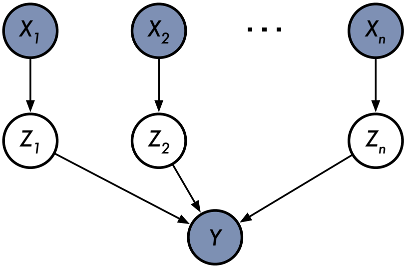
HW 5

EM for noisy-OR



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First you will show that this model is equivalent to noisy-OR.

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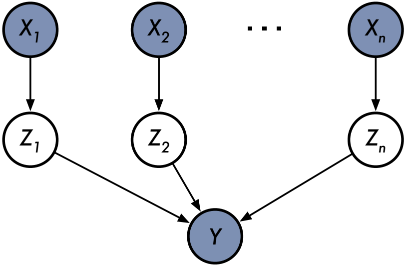

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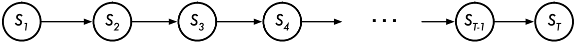
First you will show that this model is equivalent to noisy-OR.
Then you will derive the EM updates for $p_i \in [0, 1]$.

Hidden Markov Models

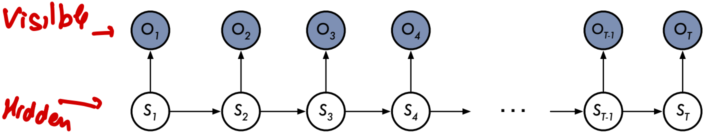
Markov Models (Review)



Two simplifying assumptions:

1. Finite Context
2. Position Invariance

Hidden Markov models (HMMs)



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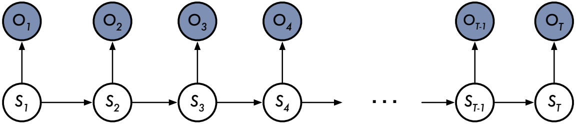

- Random variables

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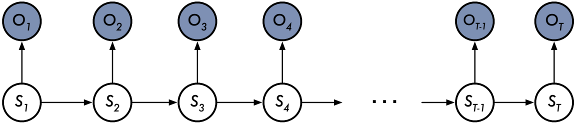

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  $S_t \in \{1, 2, \ldots, n\}$ hidden state at time t

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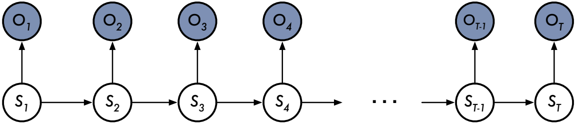

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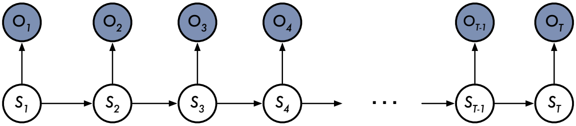

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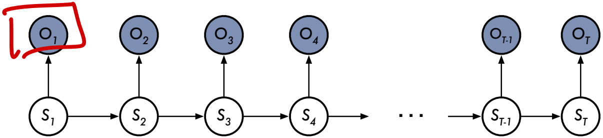

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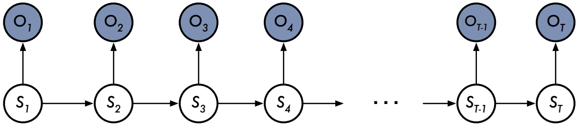
  Each observation O_t is a noisy, partial reflection of the true underlying (but hidden) state S_t of the world at time t.

Hidden Markov models (HMMs)



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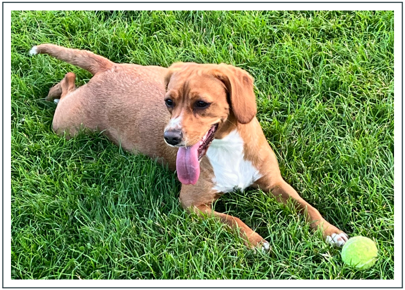
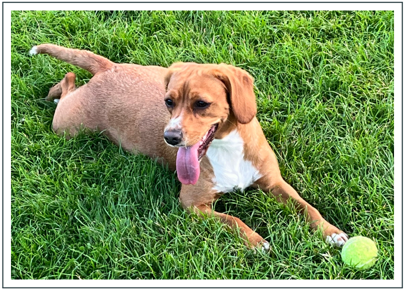
  What makes this model so useful?



This is Bubbles.
She’s an english spanador.

Housetraining a puppy



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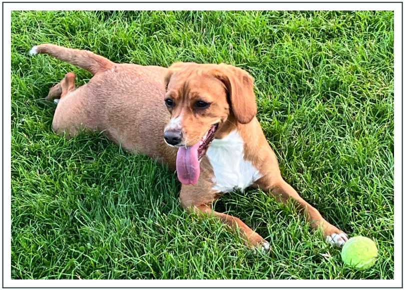
O_t

Housetraining a puppy



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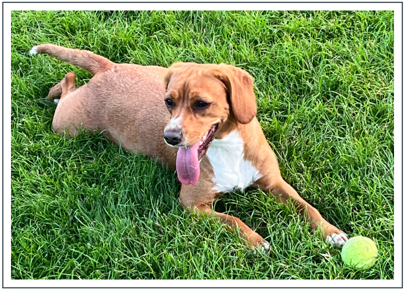
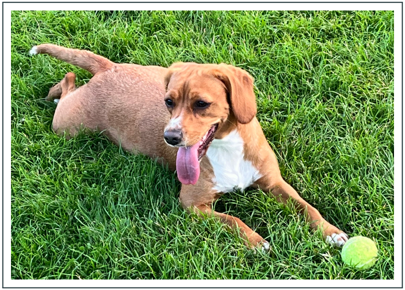
She's an english spanador.

$O_t \in \{\texttt{sleeping},$



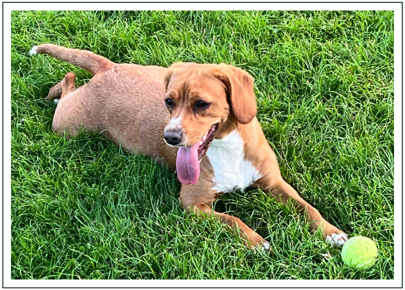
This is Bubbles.
She's an english spanador.

$O_t \in \{\texttt{sleeping}, \texttt{eating},$



This is Bubbles.
She's an english spanador.

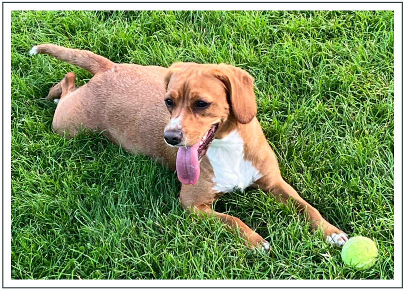
$O_t \in \{\texttt{sleeping}, \texttt{eating}, \texttt{barking},$



This is Bubbles.
She's an english spanador.

$O_t \in \{\texttt{sleeping}, \texttt{eating}, \texttt{barking}, \texttt{waiting by door}, \texttt{etc.}\}$

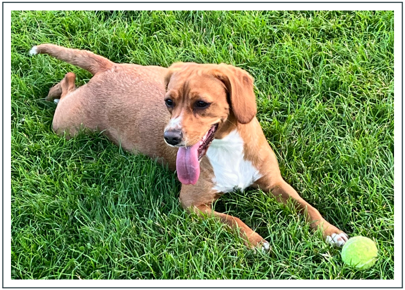
Housetraining a puppy



This is Bubbles.
She’s an english spanador.

$O_t \in \{\texttt{sleeping}, \texttt{eating}, \texttt{barking}, \texttt{waiting by door}, \texttt{etc.}\}$
S_t

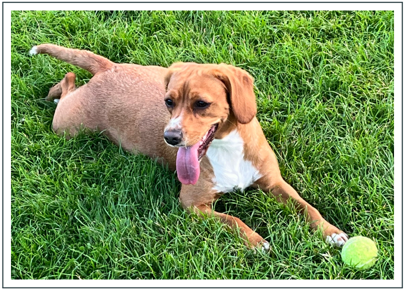
Housetraining a puppy



This is Bubbles.
She's an english spanador.

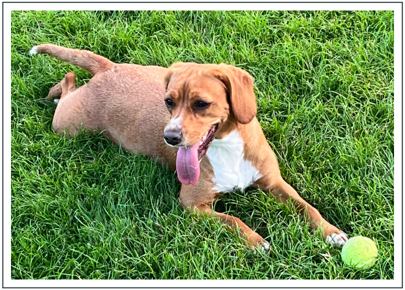
$O_t \in \{\texttt{sleeping}, \texttt{eating}, \texttt{barking}, \texttt{waiting by door}, \texttt{etc.}\}$
$S_t \in \{\texttt{playful},$

Housetraining a puppy



This is Bubbles.
She’s an english spanador.

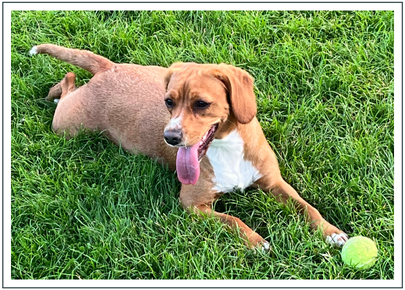
$O_t \in \{$`sleeping`, `eating`, `barking`, `waiting by door`, `etc.`$\}$
$S_t \in \{$`playful`, `hungry`,



This is Bubbles.
She's an english spanador.

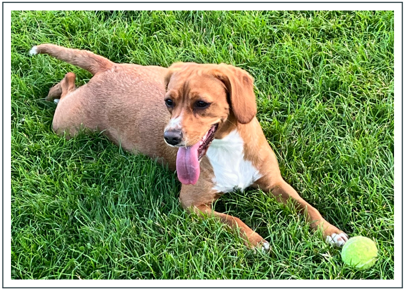
$O_t \in \{\texttt{sleeping}, \texttt{eating}, \texttt{barking}, \texttt{waiting by door}, \texttt{etc.}\}$
$S_t \in \{\texttt{playful}, \texttt{hungry}, \texttt{tired},$

Housetraining a puppy



This is Bubbles.
She's an english spanador.

$O_t \in \{\texttt{sleeping}, \texttt{eating}, \texttt{barking}, \texttt{waiting by door}, \texttt{etc.}\}$
$S_t \in \{\texttt{playful}, \texttt{hungry}, \texttt{tired}, \texttt{ready to burst}\}$



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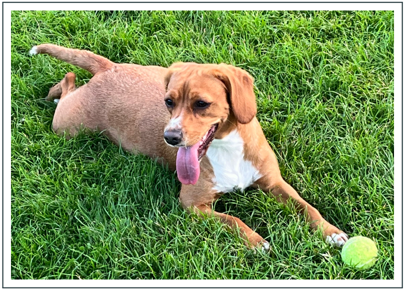
$O_t \in \{\texttt{sleeping}, \texttt{eating}, \texttt{barking}, \texttt{waiting by door}, \texttt{etc.}\}$
$S_t \in \{\texttt{playful}, \texttt{hungry}, \texttt{tired}, \texttt{ready to burst}\}$

Does she need to go outside?



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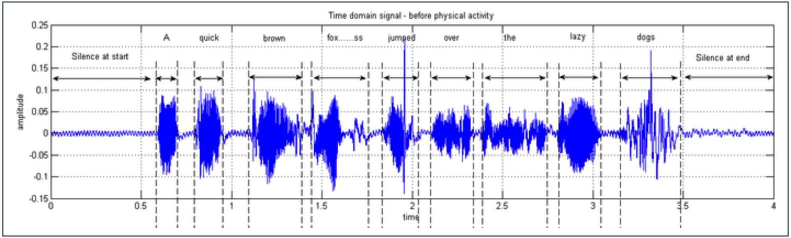
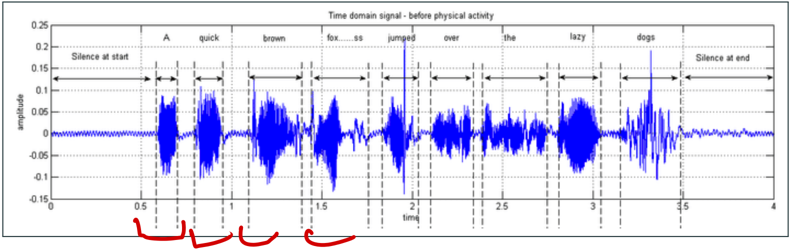
$O_t \in \{\texttt{sleeping}, \texttt{eating}, \texttt{barking}, \texttt{waiting by door}, \texttt{etc.}\}$
$S_t \in \{\texttt{playful}, \texttt{hungry}, \texttt{tired}, \texttt{ready to burst}\}$

Does she need to go outside?
What is $P(s_t | o_1, o_2, \ldots, o_t)$?

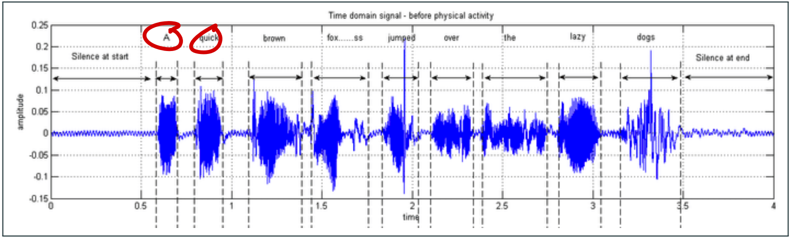

Time domain signal - before physical activity
Silence at start
A
quick
brown
fox......ss
jumped
over
the
lazy
dogs
Silence at end
amplitude
time
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
0
0.5
1
1.5
2
2.5
3
3.5
4

Speech recognition



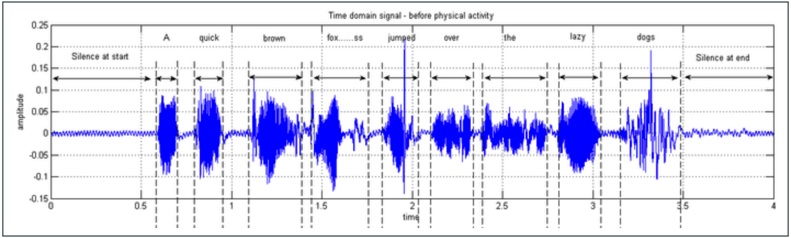
O_t is the acoustic feature vector for windowed speech at time t.

Speech recognition



O_t is the acoustic feature vector for windowed speech at time t.
S_t is the unit of language (e.g., phoneme) being uttered at time t.

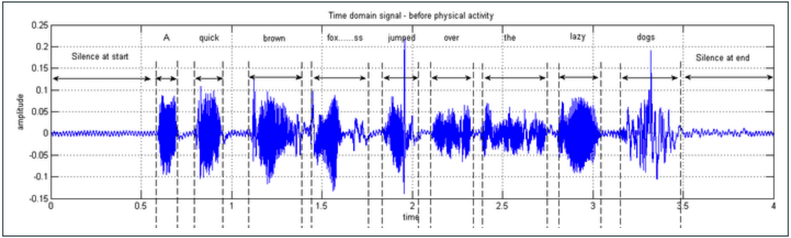
Speech recognition



O_t is the acoustic feature vector for windowed speech at time t.
S_t is the unit of language (e.g., phoneme) being uttered at time t.

What did I just hear?

Speech recognition



O_t is the acoustic feature vector for windowed speech at time t.
S_t is the unit of language (e.g., phoneme) being uttered at time t.

What did I just hear?
What is $\text{argmax}_{s_1,s_2,\ldots,s_T} P(s_1, s_2, \ldots, s_T | o_1, o_2, \ldots, o_T)$?

Indoor robot navigation

O_t encodes the sensor readings at time t.

Indoor robot navigation

O_t encodes the sensor readings at time t.

S_t encodes the robot location at time t.

Indoor robot navigation

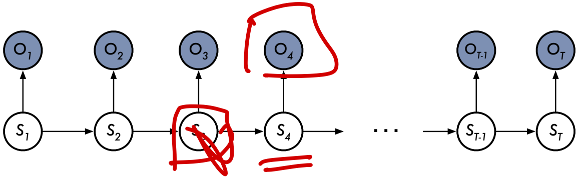
O_t encodes the sensor readings at time t.
S_t encodes the robot location at time t.

Location in the room:

Indoor robot navigation

O_t encodes the sensor readings at time t.
S_t encodes the robot location at time t.

Location in the room: what is $P(s_t|o_1, o_2, \ldots, o_t)$?



Q. Which of the following statements are True?

A. $P(S_t|S_1, S_2, \ldots, S_{t-1}) = P(S_t|S_{t-1})$

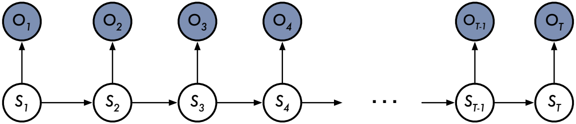
B. $P(O_t|S_1, S_2, \ldots, S_t) = P(O_t|S_t)$

C. $P(S_t|S_{t-1}) = P(S_t|S_{t-1}, O_t)$

D. A and B

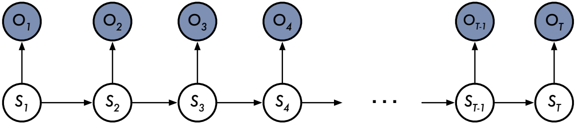
E. A, B and C

HMMs as belief networks



- Conditional independence assumptions

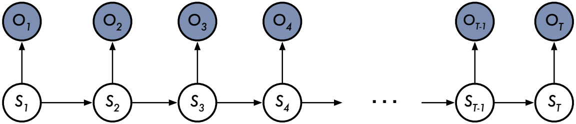
HMMs as belief networks



- Conditional independence assumptions

$$P(S_t|S_1, S_2, \ldots, S_{t-1})$$

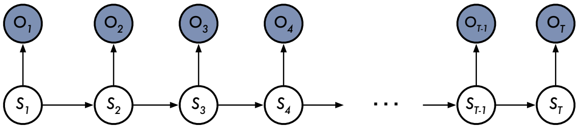
HMMs as belief networks



- Conditional independence assumptions

$$P(S_t|S_1, S_2, \ldots, S_{t-1}) \quad = \quad P(S_t|S_{t-1})$$

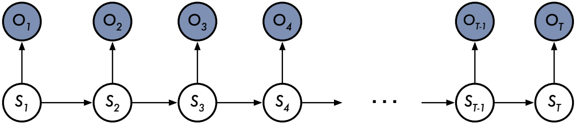
HMMs as belief networks



- Conditional independence assumptions

$$P(S_t|S_1, S_2, \ldots, S_{t-1}) = P(S_t|S_{t-1})$$
$$P(O_t|S_1, S_2, \ldots, S_T)$$

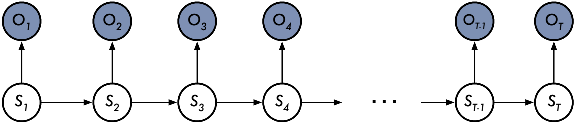
HMMs as belief networks



- Conditional independence assumptions

$$P(S_t|S_1, S_2, \ldots, S_{t-1}) = P(S_t|S_{t-1})$$
$$P(O_t|S_1, S_2, \ldots, S_T) = P(O_t|S_t)$$

HMMs as belief networks

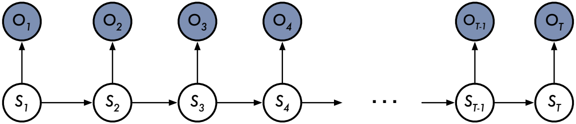


- Conditional independence assumptions

$$
\begin{aligned}
P(S_t|S_1, S_2, \ldots, S_{t-1}) &= P(S_t|S_{t-1}) \\
P(O_t|S_1, S_2, \ldots, S_T) &= P(O_t|S_t)
\end{aligned}
$$

- CPTs are shared across time

HMMs as belief networks



- Conditional independence assumptions

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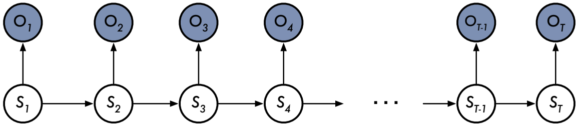
P(S_t|S_1, S_2, \ldots, S_{t-1}) &= P(S_t|S_{t-1}) \\
P(O_t|S_1, S_2, \ldots, S_T) &= P(O_t|S_t)
\end{aligned}
$$

- CPTs are shared across time

$$P(S_t = s'|S_{t-1} = s)$$

HMMs as belief networks



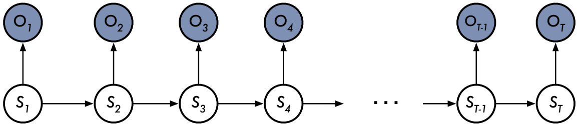
- Conditional independence assumptions

$$
\begin{aligned}
P(S_t|S_1, S_2, \ldots, S_{t-1}) &= P(S_t|S_{t-1}) \\
P(O_t|S_1, S_2, \ldots, S_T) &= P(O_t|S_t)
\end{aligned}
$$

- CPTs are shared across time

$$P(S_t = s' | S_{t-1} = s) = P(S_{t+1} = s' | S_t = s)$$

HMMs as belief networks

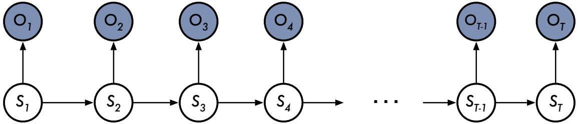


- Conditional independence assumptions

$$P(S_t|S_1, S_2, \ldots, S_{t-1}) = P(S_t|S_{t-1})$$
$$P(O_t|S_1, S_2, \ldots, S_T) = P(O_t|S_t)$$

- CPTs are shared across time

$$P(S_t = s'|S_{t-1} = s) = P(S_{t+1} = s'|S_t = s)$$
$$P(O_t = o|S_t = s)$$



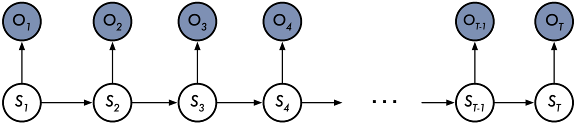
- Conditional independence assumptions

$$P(S_t|S_1, S_2, \ldots, S_{t-1}) = P(S_t|S_{t-1})$$
$$P(O_t|S_1, S_2, \ldots, S_T) = P(O_t|S_t)$$

- CPTs are shared across time

$$P(S_t = s'|S_{t-1} = s) = P(S_{t+1} = s'|S_t = s)$$
$$P(O_t = o|S_t = s) = P(O_{t+1} = o|S_{t+1} = s)$$

HMMs as belief networks



- Conditional independence assumptions

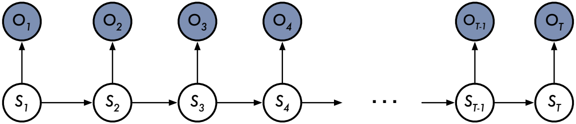
$$\begin{aligned} P(S_t|S_1, S_2, \ldots, S_{t-1}) &= P(S_t|S_{t-1}) \\ P(O_t|S_1, S_2, \ldots, S_T) &= P(O_t|S_t) \end{aligned}$$

- CPTs are shared across time

$$\begin{aligned} P(S_t = s'|S_{t-1} = s) &= P(S_{t+1} = s'|S_t = s) \\ P(O_t = o|S_t = s) &= P(O_{t+1} = o|S_{t+1} = s) \end{aligned}$$

- Joint distribution

HMMs as belief networks



- Conditional independence assumptions

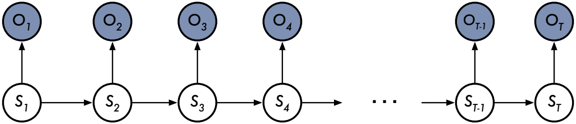
$$P(S_t|S_1, S_2, \ldots, S_{t-1}) = P(S_t|S_{t-1})$$
$$P(O_t|S_1, S_2, \ldots, S_T) = P(O_t|S_t)$$

- CPTs are shared across time

$$P(S_t = s'|S_{t-1} = s) = P(S_{t+1} = s'|S_t = s)$$
$$P(O_t = o|S_t = s) = P(O_{t+1} = o|S_{t+1} = s)$$

- Joint distribution

$P($



- Conditional independence assumptions

$$P(S_t|S_1, S_2, \ldots, S_{t-1}) = P(S_t|S_{t-1})$$
$$P(O_t|S_1, S_2, \ldots, S_T) = P(O_t|S_t)$$

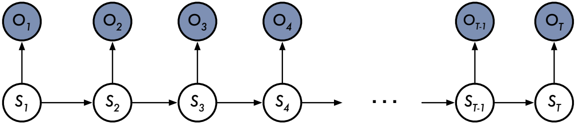
- CPTs are shared across time

$$P(S_t = s'|S_{t-1} = s) = P(S_{t+1} = s'|S_t = s)$$
$$P(O_t = o|S_t = s) = P(O_{t+1} = o|S_{t+1} = s)$$

- Joint distribution

$$P(S_1, \ldots, S_T$$

HMMs as belief networks



- Conditional independence assumptions

$$
\begin{aligned}
P(S_t|S_1, S_2, \ldots, S_{t-1}) &= P(S_t|S_{t-1}) \\
P(O_t|S_1, S_2, \ldots, S_T) &= P(O_t|S_t)
\end{aligned}
$$

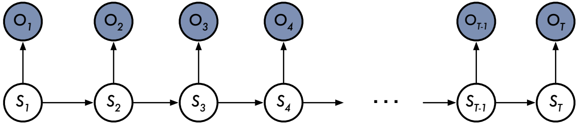
- CPTs are shared across time

$$
\begin{aligned}
P(S_t = s'|S_{t-1} = s) &= P(S_{t+1} = s'|S_t = s) \\
P(O_t = o|S_t = s) &= P(O_{t+1} = o|S_{t+1} = s)
\end{aligned}
$$

- Joint distribution

$$
P(\underbrace{S_1, \ldots, S_T}_{\vec{S}},
$$

HMMs as belief networks



- Conditional independence assumptions

$$
\begin{aligned}
P(S_t|S_1, S_2, \ldots, S_{t-1}) &= P(S_t|S_{t-1}) \\
P(O_t|S_1, S_2, \ldots, S_T) &= P(O_t|S_t)
\end{aligned}
$$

- CPTs are shared across time

$$
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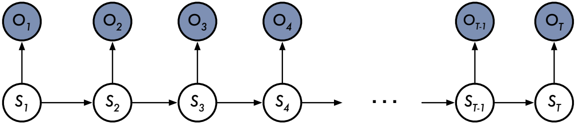
P(S_t = s'|S_{t-1} = s) &= P(S_{t+1} = s'|S_t = s) \\
P(O_t = o|S_t = s) &= P(O_{t+1} = o|S_{t+1} = s)
\end{aligned}
$$

- Joint distribution

$$P(\underbrace{S_1, \ldots, S_T}_{\vec{S}}, O_1, \ldots, O_T$$

HMMs as belief networks



- Conditional independence assumptions

$$P(S_t|S_1, S_2, \ldots, S_{t-1}) = P(S_t|S_{t-1})$$
$$P(O_t|S_1, S_2, \ldots, S_T) = P(O_t|S_t)$$

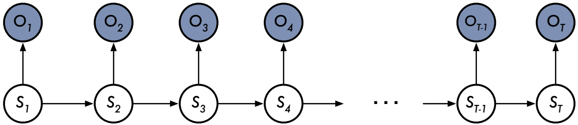
- CPTs are shared across time

$$P(S_t = s'|S_{t-1} = s) = P(S_{t+1} = s'|S_t = s)$$
$$P(O_t = o|S_t = s) = P(O_{t+1} = o|S_{t+1} = s)$$

- Joint distribution

$$P(\underbrace{S_1, \ldots, S_T}_{\vec{S}}, \underbrace{O_1, \ldots, O_T}_{\vec{O}}) =$$

HMMs as belief networks



- Conditional independence assumptions

$$
\begin{aligned}
P(S_t|S_1, S_2, \ldots, S_{t-1}) &= P(S_t|S_{t-1}) \\
P(O_t|S_1, S_2, \ldots, S_T) &= P(O_t|S_t)
\end{aligned}
$$

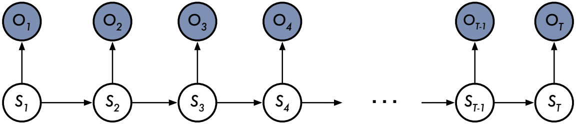
- CPTs are shared across time

$$
\begin{aligned}
P(S_t = s'|S_{t-1} = s) &= P(S_{t+1} = s'|S_t = s) \\
P(O_t = o|S_t = s) &= P(O_{t+1} = o|S_{t+1} = s)
\end{aligned}
$$

- Joint distribution

$$
P(\underbrace{S_1, \ldots, S_T}_{\vec{S}}, \underbrace{O_1, \ldots, O_T}_{\vec{O}}) = P(S_1)
$$

HMMs as belief networks



- Conditional independence assumptions

$$
\begin{aligned}
P(S_t|S_1, S_2, \ldots, S_{t-1}) &= P(S_t|S_{t-1}) \\
P(O_t|S_1, S_2, \ldots, S_T) &= P(O_t|S_t)
\end{aligned}
$$

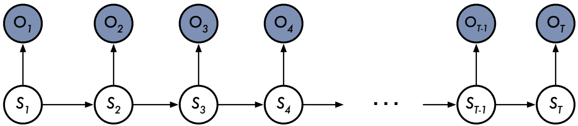
- CPTs are shared across time

$$
\begin{aligned}
P(S_t\!=\!s'|S_{t-1}\!=\!s) &= P(S_{t+1}\!=\!s'|S_t\!=\!s) \\
P(O_t\!=\!o|S_t\!=\!s) &= P(O_{t+1}\!=\!o|S_{t+1}\!=\!s)
\end{aligned}
$$

- Joint distribution

$$
P(\underbrace{S_1, \ldots, S_T}_{\vec{S}}, \underbrace{O_1, \ldots, O_T}_{\vec{O}}) = P(S_1)\, P(O_1|S_1)
$$

HMMs as belief networks



- Conditional independence assumptions

$$P(S_t|S_1, S_2, \ldots, S_{t-1}) = P(S_t|S_{t-1})$$
$$P(O_t|S_1, S_2, \ldots, S_T) = P(O_t|S_t)$$

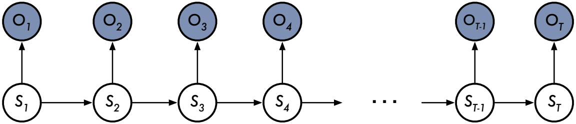
- CPTs are shared across time

$$P(S_t = s'|S_{t-1} = s) = P(S_{t+1} = s'|S_t = s)$$
$$P(O_t = o|S_t = s) = P(O_{t+1} = o|S_{t+1} = s)$$

- Joint distribution

$$P(\underbrace{S_1, \ldots, S_T}_{\vec{S}}, \underbrace{O_1, \ldots, O_T}_{\vec{O}}) = P(S_1)\, P(O_1|S_1) \prod_{t=2}^{T}$$

HMMs as belief networks



- Conditional independence assumptions

$$
\begin{aligned}
P(S_t | S_1, S_2, \ldots, S_{t-1}) &= P(S_t | S_{t-1}) \\
P(O_t | S_1, S_2, \ldots, S_T) &= P(O_t | S_t)
\end{aligned}
$$

- CPTs are shared across time

$$
\begin{aligned}
P(S_t = s' | S_{t-1} = s) &= P(S_{t+1} = s' | S_t = s) \\
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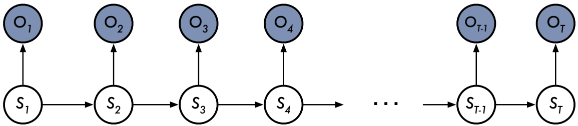
\end{aligned}
$$

- Joint distribution

$$
P(\underbrace{S_1, \ldots, S_T}_{\vec{S}}, \underbrace{O_1, \ldots, O_T}_{\vec{O}}) = P(S_1)\, P(O_1 | S_1) \prod_{t=2}^{T} \Big[P(S_t | S_{t-1})
$$

HMMs as belief networks



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$$P(S_t|S_1, S_2, \ldots, S_{t-1}) = P(S_t|S_{t-1})$$
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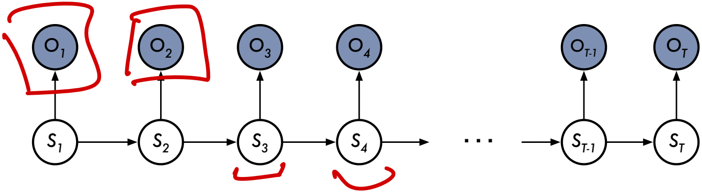
- Joint distribution

$$P(\underbrace{S_1, \ldots, S_T}_{\vec{S}}, \underbrace{O_1, \ldots, O_T}_{\vec{O}}) = P(S_1)\, P(O_1|S_1) \prod_{t=2}^{T} \left[P(S_t|S_{t-1})\, P(O_t|S_t) \right]$$

Parameters of HMMs



Q. Which of the following is NOT a parameter of the model?

A. $P(S_t|S_{t+1})$

B. $P(S_1)$

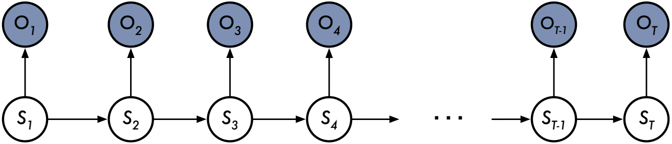
C. $P(O_t|O_{t-1})$

D. $P(O_t|S_t)$

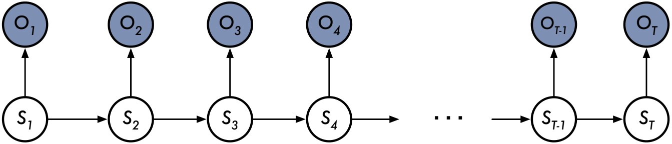
E. More than one of these is NOT a parameter of the model.

40%

Parameters of HMMs

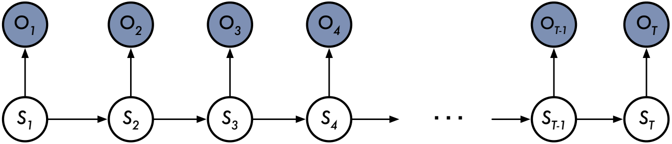


Parameters of HMMs



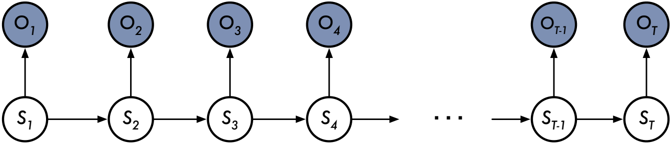
a_{ij}

Parameters of HMMs



$$a_{ij} = P(S_{t+1} = j | S_t = i)$$

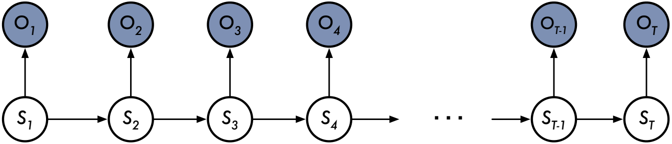
Parameters of HMMs



$$a_{ij} = P(S_{t+1} = j | S_t = i)$$

$n \times n$ **transition matrix**

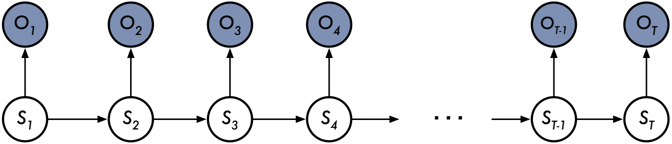
Parameters of HMMs



$a_{ij} \quad = \quad P(S_{t+1}=j|S_t=i)$ $\quad$ $n \times n$ **transition matrix**

b_{ik}

Parameters of HMMs

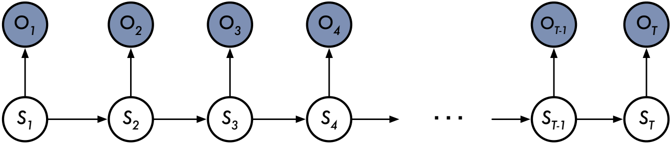


$$a_{ij} = P(S_{t+1}=j|S_t=i)$$

$n \times n$ **transition matrix**

$$b_{ik} = P(O_t=k|S_t=i)$$

Parameters of HMMs



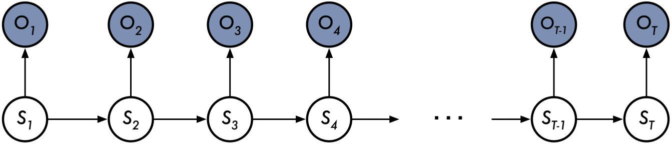
$$a_{ij} \quad = \quad P(S_{t+1} = j | S_t = i)$$

$n \times n$ **transition matrix**

$$b_{ik} \quad = \quad P(O_t = k | S_t = i)$$

$n \times m$ **emission matrix**

Parameters of HMMs



$$a_{ij} = P(S_{t+1} = j | S_t = i)$$

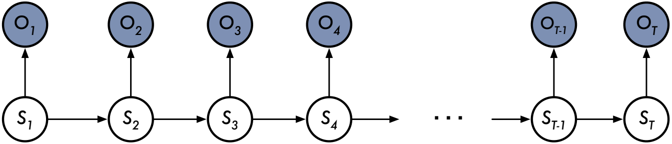
$n \times n$ **transition matrix**

$$b_{ik} = P(O_t = k | S_t = i)$$

$n \times m$ **emission matrix**

$$\pi_i$$

Parameters of HMMs



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$n \times n$ **transition matrix**

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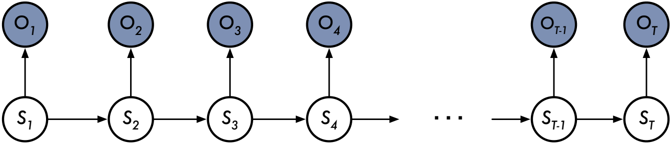
$$\pi_i = P(S_1 = i)$$

Parameters of HMMs



$$a_{ij} = P(S_{t+1}=j|S_t=i)$$

$n \times n$ **transition matrix**

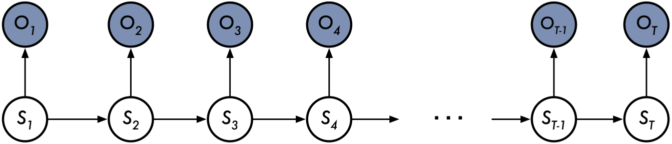
$$b_{ik} = P(O_t=k|S_t=i)$$

$n \times m$ **emission matrix**

$$\pi_i = P(S_1=i)$$

$n \times 1$ **initial state distribution**

Parameters of HMMs



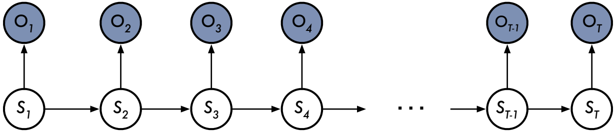
$$a_{ij} = P(S_{t+1}=j|S_t=i)$$ $n \times n$ **transition matrix**

$$b_{ik} = P(O_t=k|S_t=i)$$ $n \times m$ **emission matrix**

$$\pi_i = P(S_1=i)$$ $n \times 1$ **initial state distribution**

HMM is a polytree. **True** A **or False?** B

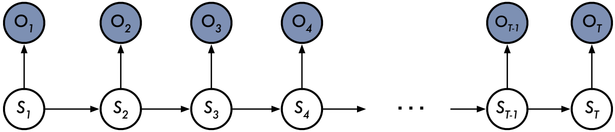
Key computations in HMMs[1]



POLYTREE!

[1] Rabiner, L. R. 1989. A tutorial on hidden Markov models and selected applications in speech recognition.

Key computations in HMMs[1]

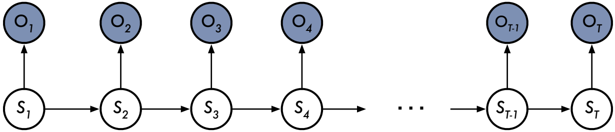


POLYTREE!

Inference

[1] Rabiner, L. R. 1989. A tutorial on hidden Markov models and selected applications in speech recognition.

Key computations in HMMs[1]



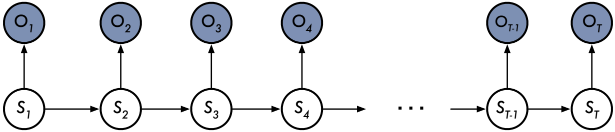
POLYTREE!

Inference

1. How to compute the likelihood $P(o_1, o_2, \ldots, o_T)$?

[1]Rabiner, L. R. 1989. A tutorial on hidden Markov models and selected applications in speech recognition.

Key computations in HMMs[1]



POLYTREE!

Inference

1. How to compute the likelihood $P(o_1, o_2, \ldots, o_T)$?
2. How to compute the most likely state sequence $\text{argmax}_{\vec{s}} P(\vec{s}|\vec{o})$?

[1]Rabiner, L. R. 1989. A tutorial on hidden Markov models and selected applications in speech recognition.

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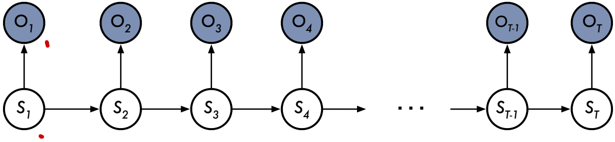

POLYTREE!

number

Inference

1. How to compute the likelihood $P(o_1, o_2, \ldots, o_T)$?
2. How to compute the most likely state sequence $\text{argmax}_{\vec{s}} P(\vec{s}|\vec{o})$?
3. How to update beliefs by computing $P(s_t|o_1, o_2, \ldots, o_t)$?

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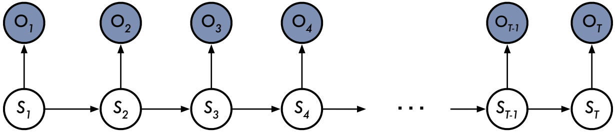

POLYTREE!

Inference

1. How to compute the likelihood $P(o_1, o_2, \ldots, o_T)$?
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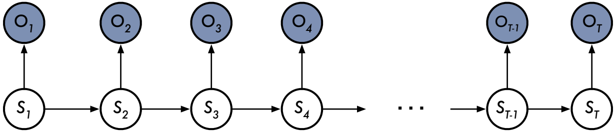
Learning

[1] Rabiner, L. R. 1989. A tutorial on hidden Markov models and selected applications in speech recognition.

Key computations in HMMs[1]



POLYTREE!

Inference

1. How to compute the likelihood $P(o_1, o_2, \ldots, o_T)$?
2. How to compute the most likely state sequence $\text{argmax}_{\vec{s}} P(\vec{s}|\vec{o})$?
3. How to update beliefs by computing $P(s_t|o_1, o_2, \ldots, o_t)$?

Learning

How to estimate parameters $\{\pi_i, a_{ij}, b_{ik}\}$ that maximize the log-likelihood of observed sequences?

[1] Rabiner, L. R. 1989. A tutorial on hidden Markov models and selected applications in speech recognition.

That’s all folks!